SNR ESTIMATION USING EXTENDED KALMAN FILTER TECHNIQUE FOR ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM) SYSTEM

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ABSTRACT

Signal to Noise Ratio (SNR) estimation of a received signal is an important and essential information for Orthogonal Frequency Division Multiplexing (OFDM) system. This is because in OFDM system, robustness in frequency selective channels can be achieved using adaptable transmission parameters. Therefore, to reckon these parameters, knowledge of SNR estimates obtained by channel state information is required for optimal performance. The performance of SNR estimation algorithm is contingent on channel estimates obtained through channel estimation schemes. In this project, two estimators which are Least Square (LS) and Minimum Mean Square Error (MMSE) estimators are simulated and analyzed. From the result obtained, LS shows better performance than MMSE in terms of Symbol Error Rate (SER) and Mean Square Error (MSE) via computer simulation. With different number of sub carriers implemented for the system model, 16, 32, 64, the result apparently shows that the SER curve of the estimator with the highest number of sub carriers, 64 is significantly lower compare with the other estimators with sub carriers of 16 and 32. Therefore, a system model which contribute to 64 sub carriers are implemented. However, in case of wireless channels, they possess non linearity where the LS and MMSE, linear estimators yield inefficient results. Therefore, to improve the SNR estimation, an efficient non linear Extended Kalman Filter (EKF) estimation, is implemented into the OFDM system. The EKF estimator outperforms the LS and MMSE estimators in terms of SER and MSE for AWGN channel. The beauty of the estimation is that it can estimate the past, present and future.

ABSTRAK

Anggaran nisbah isyarat kepada hingar (SNR) yang diterima adalah maklumat yang penting dan berguna untuk sistem Pemultipleksan Pembahagian Frekuensi ortogon (OFDM). Ini adalah kerana dalam sistem OFDM, keteguhan dalam saluran memilih frekuensi boleh dicapai dengan menggunakan parameter penghantaran yang sesuai. Oleh itu, untuk mendapatkan maklumat parameter ini, pengetahuan tentang penganggaran SNR diperolehi melalui saluran maklumat yang diperlukan untuk prestasi optimum. Prestasi algoritma anggaran SNR adalah bergantung kepada anggaran saluran yang diperolehi melalui skim anggaran saluran. Dua teknik anggaran yang telah dikaji oleh penyelidik lain telah dianalisis, *Least Square* (LS) dan *Minimum Mean Square Error* (MMSE). Daripada keputusan kajian ini, LS menunjukkan prestasi yang lebih baik daripada MMSE dari segi *Symbol Error Rate* (SER) dan *Mean Square Error* (MSE) melalui simulasi komputer. Melalui jumlah pembawa sub yang berbeza bagi model sistem, 16, 32, 64, hasilnya jelas menunjukkan bahawa keluk SER penganggar dengan bilangan pembawa sub tertinggi, 64 adalah jauh lebih rendah berbanding dengan penganggar lain dengan pembawa sub 16 dan 32. Oleh itu, satu model sistem yang menyumbang kepada 64 pembawa sub dikaji. Walau bagaimanapun, bagi kes saluran tanpa wayar, yang bukan linear, LS dan MMSE, penganggar linear menghasilkan keputusan yang tidak cekap. Bagi meningkatkan anggaran SNR dengan tepat, *Extended Kalman Filter* (EKF), dilaksanakan dalam sistem OFDM. Melalui simulasi, kaedah EKF melebihi prestasi LS dan MMSE dari segi SER dan MSE untuk saluran *Additive White Gaussian Noise* (AWGN). Kelebihan anggaran adalah bahawa ia boleh menganggarkan element lepas, semasa dan depan.

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LIST OF ABBREVIATIONS

ADSL	Asynchronous Digital Subscriber Line
AWGN	Additive White Gaussian Noise
BPSK	Binary Phase Shift Keying
CP	Cyclic Prefix
CTC	Convolutional Turbo Coding
DA	Data Aided
DAB	Digital Audio Broadcasting
DC	Direct Current
DFT	Discrete Fourier Transform
DSP	Digital Signal Processor/Processing
DVB-T	Terrestrial Video Broadcasting
EKF	Extended Kalman Filtering
FDM	Frequency Division Multiplexing
FEC	Forward Error Correction
FFT	Fast Fourier Transform
HDSL	High bit-rate Digital Subscriber Line Systems
IFFT	Inverse Fast Fourier Transform
ICI	Inter Carrier Interference
ISI	Inter Symbol Interference
KF	Kalman Filter
LS	Least Square
M_2 M_4	Second- and Fourth- Order Moments
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MSE	Mean Square Error
MMSE	Minimum Mean Square Error

NDA	Non Data Aided
NMSE	Normalised Mean Square Error
NLOS	Non Line-Of-Sight
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak-to-Average Power Ratio
PRBS	Pseudorandom Binary Sequence
PSK	Pulse Shift Keying
QPSK	Quardrature Pulse Shift Keying
QAM	Quardrature Amplitude Modulation
RS	Reed Solomon
SER	Symbol Error Rate
SFN	Single Frequency Network
SNR	Signal to Noise Ratio
SSME	Split Symbol Moments Estimator
SVR	Signal to Variance Ratio
VLSI	Very Large Scale Integrated Circuits
WiMAX	Worldwide Interoperability for Microwave Access

CHAPTER 1

INTRODUCTION

1.1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) have been invented for more than 40 years ago and is implemented in wide variety of applications in digital transmission system. OFDM has proven to be useful in frequency-selective channels to achieve high data rates and has been abundantly proposed for high-data rate transmission system. In the past, OFDM applications have been scarce because of the complexity involved in practical implementation. Recent advances in technology such as Very Large Scale Integrated Circuits (VLSI) and Digital Signal Processors (DSPs) have enabled the cost-effective and practical implementation of the Discrete Fourier Transform (DFT) / Inverse Fast Fourier Transform (IFFT) via the Fast Fourier Transform (FFT) / Inverse Fast Fourier Transform (IFFT) operation on a single chip for Digital Audio Broadcasting standard (DAB) and TerrestrialVideo Broadcasting (DVB-T) systems. OFDM is also employed for fixed wireline applications such as Asynchronous Digital Subscriber Line (ADSL) and High bit-rate Digital Subscriber Line Systems (HDSL). Standard IEEE 802.11a/g, wireless broadband access standards IEEE 802.16a (WiMAX), and the core technique for fourth-generation (4G) wireless mobile communications were also incorporate OFDM.

OFDM is a multicarrier modulation scheme with excellent robustness in multi-path environments by dividing the high-data rate channel over multiple and very narrow parallel channels,where each is modulated on a different subcarriers. The data rate in the individual carriers are reduced significantly, while still maintaining overall throughput. To ensure that sub-carriers do not interfere with each other in frequency domain, subchannels orthogonalization is performed using the fast Fourier transform (FFT). Hence the inter symbol interference (ISI) due to multipath delay spread is reduced. Pilot symbols are used to estimate the complex gains of the parallel subchannels. Moreover, cyclic prefix added at the transmitter helps to reduce the receiver complexity for FFT processing. This mechanism facilitates the design of single frequency networks (SFNs). However, reliable communication in wireless system, becomes a difficult task as the transmitted data is not only corrupted by Additive White Gaussian Noise (AWGN), but also suffers from ISI as well as interference or colored noise from other users. Color of the noise is caused by its uneven spectral variation. Accurate Signal to Noise Ratio (SNR) estimation at receiver is important to modify the transmission parameter for channel quality control.

SNR can be used to characterize the link quality. It is an important factor in a receiver performance on how well the recovering of the information-carrying signal from its corrupted version and hence how reliably information can be communicated. SNR can be defined in its simplest form as ratio of signal power to noise power. Generally, there are two categories of knowledge-based SNR estimators, Data-aided (DA) and Non Data-Aided (NDA). DA estimators are based on the knowledge of the transmitted data, in which the bandwidth efficiency is reduced, while NDA estimates SNR from an unknown information.

1.2 Problem of Statement

Modern wireless communication systems has come a long way since 1897. It has the ability transfer of information between two or more points without the need of a fixed cable connection. Wireless communication systems is used to meets many needs, from the traditional voice application to multimedia, pushing the needs for higher data rate and faster transmission speed. As the demand of data rate and transmission speed increases in recent years, the noise and ISI also increases due to multipath fadings.

Multipath signals due to different delays is caused by reflections of the signal and different propagation paths between the transmitters and receivers. These reflected components add up either constructively or destructively at the receiver, and the resulting effect is known as frequency selective fading. During the propagation of the signal through the channel, the signal also suffers rapid fluctuation in amplitude and phase. It is very important to combat the effects of the fadings because the amount of fading is directly related to the throughput of the system. A multicarrier modulation, OFDM is a popular and powerful method to combat the frequency selective channels. A fixed-mode transceivers cannot perform effectively over a dispersive channel, as severe SNR fluctuations occurs. It is this drawback that been a motivating factor for research on adaptive transceivers. Adaptation in conjunction with OFDM, is a powerful method to mitigate the frequency selective fading and utilize the maximum available capacity of the time varying channel. The performance of adaptive OFDM systems depends on the effectiveness of techniques used in channel quality estimation, particularly SNR estimation. In order to achieve improvement by exploiting frequency selective channels in designing OFDM system, accurate and exact SNR estimation is requisite. There are many other applications that can exploit SNR information, such like channel estimation through interpolation and optimal soft information generation for high performance decoding algorithms.

1.3 Project Objectives

The objectives of the project is to:

(i) Analyse different kinds of SNR estimation and their performance in OFDM systems.

(ii) Develop the SNR estimation for OFDM receiver systems using Matlab / Simulink environment.

(iii) Evaluate the performance of the developed SNR estimation in terms of Symbol Error Rate (SER) and Mean Square Error (MSE).

1.4 Scopes

In this section, SNR estimation in OFDM system is proposed. The proposed system is to be simulated in Matlab / Simulink environment. Parameters of the simulated system [1] is proposed as follows:

Table 1.1 Parameter of OFDM systems

Parameter	Values
Channel Bandwidth (MHz)	5
Sampling Frequency (f_p, MHz)	5.6
IFFT Size	512
Number of sub carriers	128
Number of sub bands	32
Number of sub carriers per sub band	16
Frame size	6
SNR	1-35dB
Modulation schemes	MQAM
Carrier Frequency (GHz)	2
Subcarrier Frequency Spacing (kHz)	10.94
Useful Symbol Time ($T_b = 1/f$, µs)	91.4
Guard Time Duration ($T_g = T_b/8$, µs)	11.4
OFDM Symbol duration ($T_s = T_b + T_g$, µs)	102.9
Number of OFDM Symbols (5 ms Frame)	48

1.5 Outline

This thesis comprises of six chapters.

Chapter 1:
This chapter introduces the project overview, which consists of OFDM and SNR estimation introduction, problem of statement, objectives, scope and outline.

Chapter 2:
This chapter presents a detailed description of OFDM systems, which consists of the building blocks, operation, and signal processing.

Chapter 3:

This chapter outlines a brief overview of works done by previous researchers which describes the enabling techniques that facilitate efficient SNR estimation. This chapter also discuss in details on the consideration of wireless channel models, with white and Gaussian distributed interfering noise, AWGN and fadings.

Chapter 4:

This chapter presents the MATLAB/SIMULINK project model simulation discussion. This chapter will also give a brief explanation and description for each model block.

Chapter 5:

In this chapter, the simulation results on the model performance are illustrated. This is followed with the simulation result analysis in terms of SER and MSE.

Chapter 6:

This chapter presents the conclusions and topics for further research, of this work.

CHAPTER 2

SNR ESTIMATION : A REVIEW

2.1 Introduction

This chapter discuss related research work focusing on SNR estimation algorithms and channel estimation schemes done by other researchers.

2.1.1 System Model

SNR estimation in OFDM systems which utilize LS estimation schemes is proposed. Figure 2.1 shows frame structure where data symbols preceded by set of preambles containing known information for channel estimation and synchronization purposes.

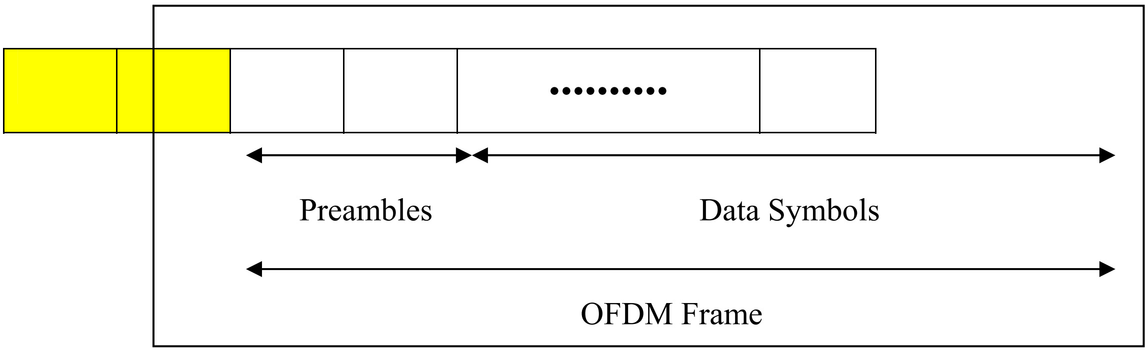

Figure 2.1 OFDM frame structure

Assume N preambles, each preamble composed of n modulated subcarriers. $C(i,j)$ represent complex data symbol on j^{th} subcarrier in i^{th} preamble. According to

OFDM standards as preambles consist of QPSK modulated subcarriers, assume unit magnitude modulated subcarrier $|C(i,j)|^2 = 1$. Perfect synchronization is assumed in frequency domain and the received signal is given as

$$Y(i,j) = \sqrt{S}C(i,j)H(i,j) + \sqrt{W}\eta(i,j) \tag{2.1}$$

where S, W are transmitted signal and noise powers respectively, $\eta(i,j)$ is sampled complex zero mean AWGN and $H(i,j)$ is channel frequency response expressed as

$$H(i,j) = \sum_{l=0}^{L-1} h(\tau_l + iT_s)e^{-j2\pi\frac{nT_l}{T_s}} \tag{2.2}$$

where L denote the length of Channel Impulse Response (CIR), $h(\tau_l + iT_s)$ signify channel path gain with delay τ_l in l^{th} path of i^{th} OFDM preamble of duration T_s. Consider channel to be constant for whole frame assuming that SNR estimation algorithms are implemented for adaptive transmissions. CIR paths are opted to be integer multiples of system sampling rate where $h(l) = h\left(l\frac{T_s}{N}\right) = h(\tau_l)$. The estimates of average SNR and SNR per subcarrier are valid for all data carrying OFDM symbols.

As shown in [2], the average SNR is given as

$$\rho_{av} = (E\{1/N \sum_{(j=0)}^{(n-1)} |\sqrt{S}\, C(j)H(j)|^2\})/(E\{1/N \sum_{(j=0)}^{(n-1} \tag{2.3}$$

The SNR on the j^{th} subcarrier is expressed as

$$\rho(j) = (E\{|\sqrt{S}\, C(j)H(j)|^2\})/(E\{|\sqrt{W}\, \eta(j)|^2\}) = \rho_{av}\, |H(j)|^2 \tag{2.4}$$

2.2 SNR estimation algorithms

SNR has long been used as the standard measure of analog signals quality in noisy environments. There are various types of SNR estimation methods developed for OFDM systems usage as shown in Figure 2.2.

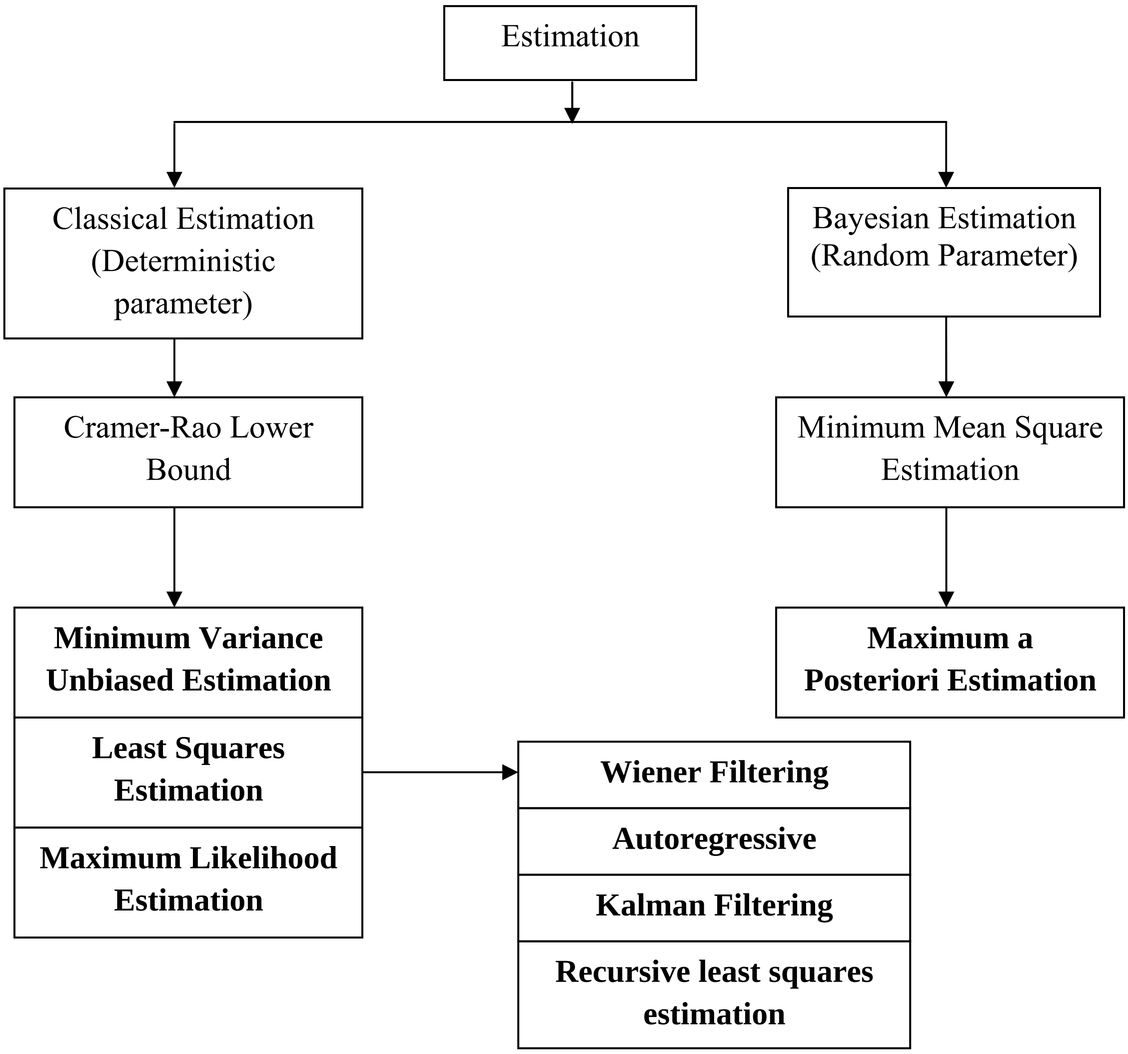

Figure 2.2 Estimation methods

Pauluzzi D.R. [3] present five different SNR estimation techniques for Pulse Shift Keying (PSK) modulation in an AWGN channel. The algorithms are called SSME (Split Symbol Moments Estimator), which is valid for Binary Pulse Shift Keying (BPSK) modulation only, ML (Maximum Likelihood), SNV (Signal to Noise Variance), M_2M_4 (Second- and Fourth-Order Moments) and SVR (Signal to

Variance Ratio). ML estimator is implemented as NDA and DA estimators where DA achieves better results at low SNR values. SNV estimator is a special case of ML estimator which is based on first and second order moments of the sampled output. Second-and-Fourth-Order Moments (M_2M_4) estimator shows similar performance as ML estimator at high SNR values. SVR estimator performs similar to ML at low SNR values. The performance of the SNR estimator only applies if the transmitter knows the channel estimates. [3] Pauluzzi D.R. also assumes that in the *jth* symbol period, the *ith* pilot subcarrier is modulated with a complex value $a(i,j)$. The same pilot signal is assumed to be sent on the same pilot subcarrier in different OFDM symbol periods, which means $a(i,j) = a(i,l)$ for any i,j and l. Then the complex baseband system model for the *ith* pilot subcarrier can be formulated as

$$y(i,j) = \sqrt{S}h(i,j)a(i,j) + \sqrt{N}n(i,j) \tag{2.5}$$

where $n(i,j)$ is complex, zero-mean AWGN and $h(i,j)$ is the complex channel factor. It is assumed that the variance of $h(i,j)$, $n(i,j)$ and $a(i,j)$ are assumed to be normalized to unity. S is a signal power scale factor, and N is a noise power scale factor. OFDM converts a multipath channel into a set of parallel time-variant linear channels.

The M_2M_4 algorithm and Bourmard's Algorithm [4] does not require the channel estimates in order to estimate the SNR estimation at the back-end of receiver. SNR estimation algorithm for 2X2 Multiple Input Multiple Output (MIMO)-OFDM systems in [5] requires 30 preambles at each transmitting antenna for noise estimates. The performance of the algorithm deteriorates with faster channel fading as Doppler frequency increases. This algorithm is based on the assumption that the channel varies slowly in time as well as frequency. Two consecutive time-domain channel estimates for any antenna pair are considered identical. In addition the channel degradation for adjacent OFDM subcarriers is considered the same. In this technique, SNR estimation is obtained from noise variance and Least Square (LS) channel estimates. Many of these estimators estimates at the back-end of the received. Only a few SNR estimations at front-end of the receiver are proposed. Rana Shahid Manzoor and et. [6] proposed noise power and SNR estimation autocorrelation-based using only one OFDM preamble at front-end of the receiver and data-aided linear prediction based SNR estimation for

AWGN channel. It is proven that the NMSE performs better result with 1 OFDM preamble compared to 30 and 50 OFDM preambles by Boumard, S. [4] and Reddy, S. [7]. It is observed that front-end estimation of SNR is performing excellently and providing reliable estimation by Rana Shahid Manzoor and et. [6].

Many of these estimators are based on the knowledge of pilot sequence, Data-Aided (DA) estimation. DA based approaches are widely implemented to estimate the channel characteristics and to correct the corrupted channel due to multipath fading. The SNR estimation technique presented by Xiadong, X. [8], data-aided estimator is based on tracking the delay-subspace using the estimated channel correlation matrix. This method required M pilot subcarriers to be inserted into every OFDM symbol. The channel frequency response for each pilot was found when the received pilot was divided with the original. The correlation matrix R were computed through eigenvalue decomposition and L, number of paths, was estimated by the Minimum Descriptive Length technique in [9]. The estimated eigenvalues and number of paths was then used to compute the SNR. It was shown by simulation results, that this estimator is able to estimate the true SNR accurately after an observation interval of about 20 OFDM symbols for various fading channels. However, in practical wireless system, the assumption that $M > L$ made by M. Wax [9] may not apply.

DA SNR estimators using training sequence limits system through-out. In order to overcome DA method limitation, NDA SNR estimator is proposed in [2] where the technique is targeted towards applications such as cognitive radio. This is because the terminals need to sense the link quality with all the surrounding networks in order to find the most suitable link for communication. This method does not require the receiver to know the pilots' locations. Instead the cyclostationarity induced by the cyclic-prefix is used to determine the SNR. They often suffer from high computation complexity and low convergence speed since they often need a large amount of receiving data to obtain some statistical information which is the cyclostationarity induced by the cyclic prefix. Blind channel estimation methods are not suitable for applications with fast varying fading channels. Table 2.1 shows the summary of the previous works by other researchers.

Table 2.1 Comparison of previous works by other researchers.

No.	Author & Year	Techniques	Remarks
1.	Pauluzzi D.R & Norman C.B. (2005)	-SSME, ML, SNV, M_2 M_4 & SVR	✓ ML (DA achieves better results at low SNR values) ✓ SNV is a special case of ML based on 1st and 2nd order of sampled output. ✓ M_2M_4 = ML at high SNR values. ✓ SVR = ML at low SNR values
2.	Reddy, S. & Arslan H. (2003)	50 OFDM preamble	
3.	Boumard, S. (2003)	Noise variance & LS channel estimates, 30 OFDM preamble	✓ The performance deteriorates with faster channel fading. ✓ Assumption : Channel varies slowly in time & frequency
4.	Rana Shahid Manzoor, Wabo Majavu, Varun Joeti, Nidal Kamel & Muhammad Asif (2007)	Autocorrelation-based estimation, 1 OFDM preamble	✓ NMSE performs better result with 1 OFDM preamble compared to more OFDM preambles.
5.	Xiadong X., Ya Jing & Xiaohu Y.	DA estimator based on tracking the delay-subspace using the estimated channel correlation matrix	✓ Estimated eigenvalues & number of paths, L was used to compute SNR. ✓ Able to estimate the true SNR accurately after an observation interval of about 20 OFDM symbols.
6.	M. Wax & T. Kailath	Minimum Descriptive Length	✓ Assumption that pilot subcarriers, M > number of path, L is impractical in wireless system.
7.	Socheleau, F., Abdeldjalil Aissa-El-Bey & Houcke, S.	Cyclostationarity induced by CP	✓ Do not rely on any knowledge of the modulation symbols. ✓ Do not waste bandwidth ✓ Suffer from high computation complexity ✓ Need large amount of receiving data to obtain statistical information.

2.2.1 Minimum Mean Square Error (MMSE) Estimation

The MMSE estimator describes the approach which minimizes the MSE and has high complexity. With a set of N independent Gaussian channels as shown in Figure 2.3, the system can be written in

$$y_k = h_k x_k + n_k, \qquad k=0,\dots,N-1 \tag{2.6}$$

where h_k is the complex channel attenuation.

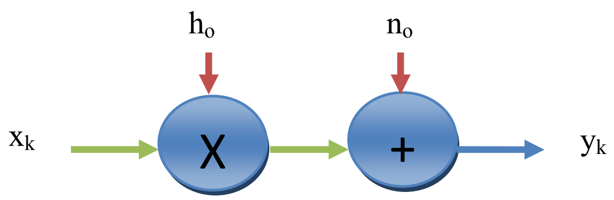

Figure 2.3 Gaussian channels

(2.6) can be rewritten in matrix notation

$$y = XFg + n \tag{2.7}$$

where X is a matrix with the elements of x on its diagonal and

$$F = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \tag{2.8}$$

is the DFT-matrix with

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}} \tag{2.9}$$

If the channel vector g is Gaussian and uncorrelated with the channel noise n, the MMSE estimate of g becomes [10]

$$\hat{g}_{MMSE} = R_{gy} R_{yy}^{-1} y \tag{2.10}$$

where

$$R_{gy} = E\{gy^H\} = R_{gg}F^H X^H \tag{2.11}$$

is the cross covariance matrix between g and y, R_{gg} is the auto-covariance matrix of g and

$$R_{yy} = E\{yy^H\} = XFR_{gg}F^H X^H + \sigma_n^2 I_N \tag{2.12}$$

is the auto-covariance matrix of y, σ_n^2 denotes the noise variance $E\{|n_k|^2\}$.
Since the columns in F are orthonormal, frequency-domain MMSE, $\hat{h}_{MMSE}$ can be generated by $\hat{g}_{MMSE}$.

$$\hat{h}_{MMSE} = \hat{F} g_{MMSE} = F Q_{MMSE} F^H X^H y \tag{2.13}$$

where

$$Q_{MMSE} = R_{gg}\left[\left(F^H X^H X F\right)^{-1}\sigma_n^2 + R_{gg}\right]^{-1}\left(F^H X^H X F\right)^{-1} \tag{2.14}$$

2.2.2 Least Square (LS) Estimation

Least Square (LS) method is about estimating parameters by minimizing the squared discrepancies between observed data and their expected values. LS estimation method attains low computational complexity, and it is easy to implement because it does not require optimization procedures in its computation. The method is used to solve imprecisely defined system [11]. The LS estimate of the attenuations h, given the received data Y and the transmitted symbol X is [12].

$$\hat{h}_{ls} = X^{-1}Y = \left[\frac{y_0}{x_0}\ \frac{y_1}{x_1} \dots \frac{y_{N-1}}{x_{N-1}}\right]^T \tag{2.15}$$

This is reduced from

$$\hat{h}_{ls} = F Q_{LS} F^H X^H y \tag{2.16}$$

where

$$Q_{LS} = (F^H X^H X F)^{-1} \tag{2.17}$$

However, LS estimator has some limitations. It has high mean square error which unable to give reasonable or correct estimates at all times. Since LS estimation is a linear estimation scheme, it is not appropriate for the non Gaussian channel, where in reality wireless channels do posses some non linear characteristics which need to be considered for processing.

2.2.3 Kalman Filter Estimation

Kalman filter is the optimal estimate for linear system system models. In 1960, **R.E Kalman** published his famous paper describing a recursive solution to the discrete-data linear filtering problem. The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, which minimizes the mean of the squared error [13]. It is very powerful in several aspects: it supports estimations of past, present, and even future states, and even when the nature of the modeled system is unknown.

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of noisy measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projected forward the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback for incorporating a new measurement into the a priori estimate to obtain an improved a posterior estimate.

The time update equation is also known as predictor equations, while the measurement update equations can be thought of as corrector equations. The final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems as shown in Figure 2.3. It shows the ongoing discrete Kalman filter cycle. The time update projects and the current state estimate ahead in time.

The measurement update adjusts the projected estimate by an actual measurement at that time.

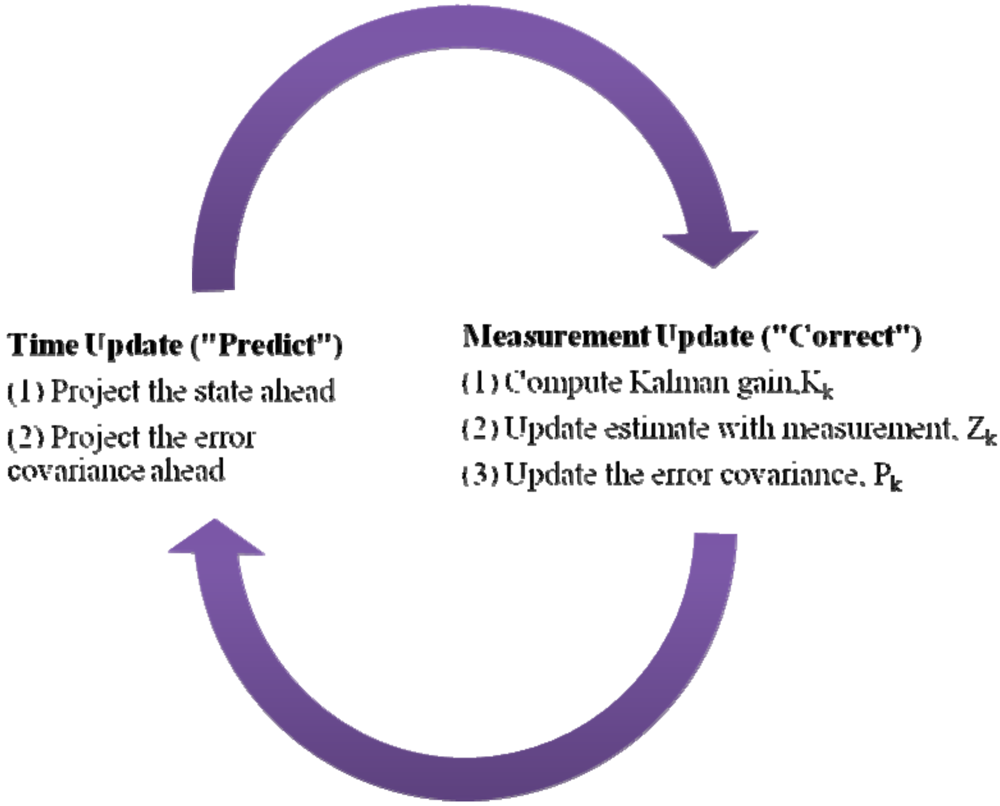

Figure 2.4 The Kalman filter cycle

The first task during the measurement update is to compute the Kalman gain, K_k. The next step is to actually measure the process to obtain z_k, and then to generate a posterior state by incorporating the $\hat{y}_k$ measurement. The final step is to obtain a posterior error covariance estimate, P_k.

After each time and measurement update pair, the process is repeated with the previous posterior estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter.

The following equations in Table 2.2 presents the time and measurement updates.

Table 2.2 Discrete Kalman filter time and measurement update equations.

Discrete Kalman filter time update equations	
$\hat{y}_k^- = A\hat{y}_{k-1} + Bu_{k-1}$	$\hat{y}_k^-$, (minus) shows the priori state estimate at step k. $\hat{y}_k = E[\hat{y}_k]$ is the posterior state estimate at step k
$P_k^- = AP_{k-1}A^T + Q$	Q = process noise covariance $P_k^- = E[e_k^- e_k^{-T}]$, a priori estimate error covariance where $e_k^- = y_k - \hat{y}_k^-$ (a priori estimate errors) and $e_k = y_k - \hat{y}_k$ (a posterior estimate errors)

Discrete Kalman filter measurement update equations	
$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$	K_k = gain or blending factor that minimizes the a posterior error covariance, P_k R = measurement noise covariance
$\hat{y}_k = \hat{y}_k^- + K_k(z_k - H\hat{y}_k^-)$ $z_k = Hy_k + v_k$, Where v_k= measurement noise	$(z_k - H\hat{y}_k^-)$ is called the measurement innovation, or residual. The residual reflects the discrepancy between the predicted measurement, $H\hat{y}_k^-$ and the actual measurement z_k. A residual of zero means that the two are in complete agreement.
$P_k = (I - K_k H)P_k^-$	$P_k = E\left[e_k e_k^T\right]$, a posterior estimate error covariance where $e_k = y_k - \hat{y}_k$

CHAPTER 3

AN OVERVIEW OF OFDM

3.1 Introduction

In this chapter, the basics of OFDM are discussed. In particular, a general description and a brief history of OFDM are presented. In addition, the OFDM block diagram, operation and signal processing are also explained briefly.

3.2 Overview of OFDM

OFDM technology was introduced back to the middle 60s. R.W. Chang [14] proved that multiple data streams can be transmitted through a linear band limited multi-channel without the ISI. The transmission of a band limited signal was synthesized on multi-channel. For further improvement to suppress ISI, S.B. Weinstein [15] made major contribution to OFDM in 1971 on how to modulate/demodulate band signal by Discrete Fourier Transformation (DCT). Empty guard interval between two adjacent symbols was proposed, but the orthogonality between two subcarriers over a frequency selective channel cannot be ensured. To ensure the orthogonality among subcarriers of an OFDM symbol, A. Peled [5] introduced the concept of cyclic prefix (CP) in 1980. The CP is copied from the end of the OFDM symbol and was transmitted followed by each OFDM symbol. Inter-carrier interference (ICI) can be avoided when the length of CP is larger than the impulse response of the fading channel.

OFDM is an efficient high data rate transmission technique for wireless communication. It is a combination of multi-carrier modulation and multiplexing techniques with high bandwidth efficiency and robustness in multipath and fading environments where OFDM system is more resilient in Non Line-Of-Sight (NLOS) environment. This is because of the equalization is done on a subset of sub-carriers instead of a single broader carrier.

In OFDM, the communication system divides a wide radio channel into several narrow sub-channels and data is transmitted in parallel on these sub-channels on different frequencies or sub-carriers. Therefore, by creating the N parallel sub-channels, the bandwidth of the modulation symbol are also by the factor of N. The summation of all the individual sub-channels data rates will result in total desired symbol rate, with the drastic reduction of the ISI distortion. The sub-carriers are closely spaced to each other without causing interference, removing guard bands between adjacent sub-carriers because the frequencies (sub-carriers) are orthogonal (independent of each other). This means that the peak of one sub-carrier coincides with the null of an adjacent sub-carrier as shown in Figure 3.1. The orthogonality between sub-carriers prevents the demodulators from seeing frequencies other than their own.

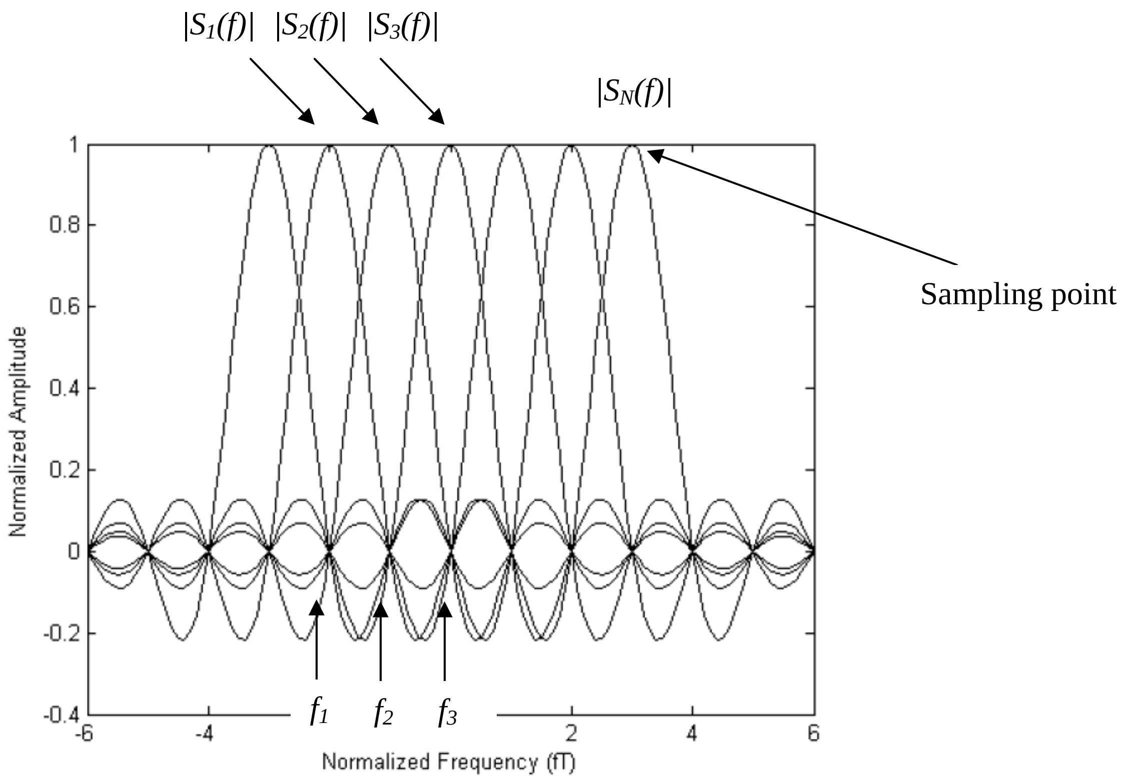

Figure 3.1 The frequency division for OFDM systems

As seen, the spectra of the sub-carriers are not completely separated, but overlap to some degree. With the so called orthogonality related method, it is the reason why the information transmitted over the carriers can still be separated. From the Figure 3.1 also, each sub-carrier is represented by a different peak. In addition, the peak of each sub-carrier corresponds directly with the zero crossing of all channels. To preserve perfect orthogonality, certain conditions need to be satisfied. The receiver and the transmitter must be perfectly synchronized, which means they both must assume exactly the same modulation frequency and the same time-scale for transmission. A more important condition is that there should absolutely be no multipath, which is solved by cyclically extending the symbol by a guard interval, and the explanation of which is given in the following sections.

3.2.1 Basic Principles of OFDM System

A block diagram of a basic OFDM system is illustrated in Figure 3.2.

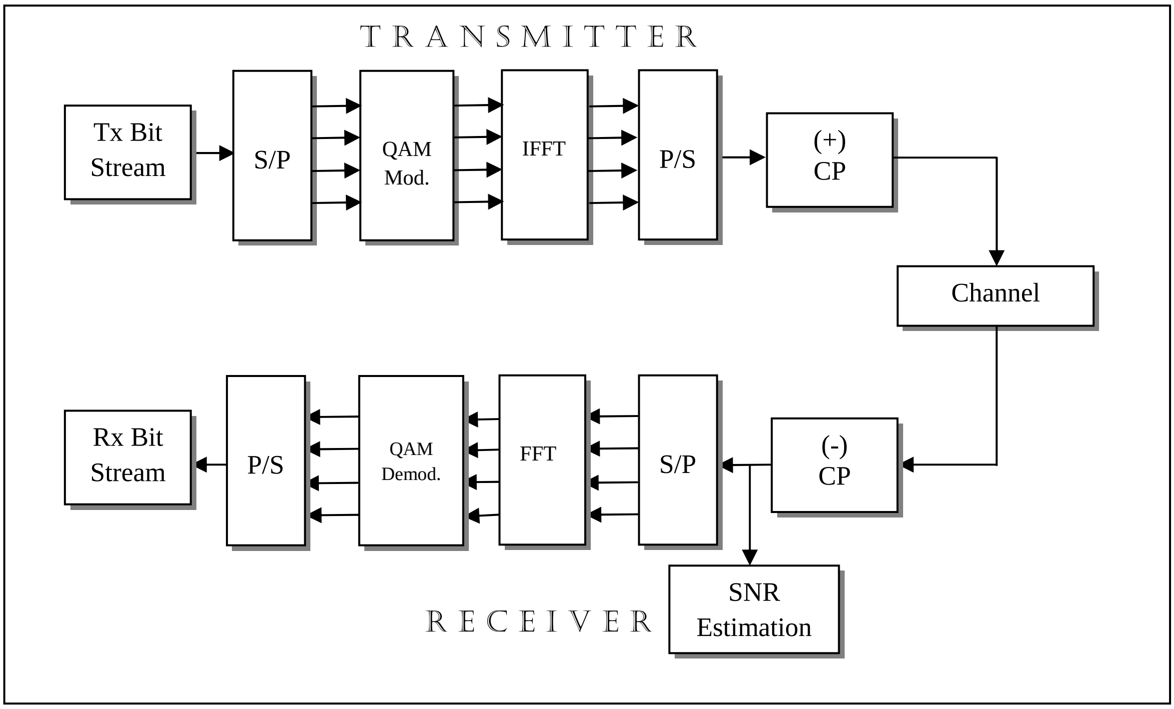

Figure 3.2 OFDM system

For high speed communications, principle of OFDM posits that frequency selective channel is evenly divided into N frequency flat subchannels. At the transmitter, random bits of data are coded and interleaved before modulation. The bit stream is split into multiple parallel streams, which reduces the bit rate using serial-to-parallel (S/P) converter. These bit streams lie in the frequency domain, so that each element in vector bit streams is assigned to one subchannel. The generated parallel data streams are modulated using QAM signal constellation to map them individually on each subcarrier. IFFT is applied to these parallel bit streams to obtain the time domain OFDM symbols. IFFT is used in OFDM system to infix the orthogonal property between the subcarriers. The N subcarriers are transformed into N point IFFT, with time domain representation of IFFT written as:

EMBED Equation.3

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k).e^{\frac{j2\pi nk}{N}} \qquad (3.1)$$

where *X(k)* is the symbol transmitted on the k^{th} subcarrier and N is the total number of subcarriers. Cyclic prefix (CP) is added to the transmitted symbol to avoid ISI before the signal is transmitted.

At the receiver, the transmitted bit stream accumulates multipath fading effects of channel. As the CP abides redundant information, removing it reduces the complexity for FFT. The received signal is transformed into frequency domain using N point FFT which can be given as

EMBED Equation.3

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x(n).e^{\frac{-j2\pi nk}{N}} \qquad (3.2)$$

X(k) on FFT is demodulated at N subcarrier frequencies relying on N demodulators to convert them into parallel bit streams. The parallel bit streams are then converted to serial bit stream using parallel-to-serial (P/S). The symbols are finally decoded to obtain the transmitted information bits. A detailed system explanation of the function for each block in Figure 3.2 is presented in the following sections.

3.2.2 Serial to Parallel Conversion

The data symbols are divided onto N parallel sub-carriers. This makes the symbols on each sub-carrier N times longer than its serial counterpart. The effect of a time dispersive channel is reduced by making the symbol duration longer than the maximum excess delay of the channel. Each parallel data stream is modulated onto a sub-carrier at a unique frequency and combined with the other sub-carriers to produce a serial stream of transmission data. Proper selection of transmission parameters can greatly reduce, if not eliminate ISI because the delay spread will be less than the symbol duration.

3.2.3 Modulation

Binary data from a memory device or from a digital processing stream is used as the modulating (baseband) signal. The following steps may be carried out in order to apply modulation to the carriers in OFDM:

- Combine the binary data into symbols according to the number of bits/symbol selected.
- Convert the serial symbol stream into parallel segments according to the number of carriers, and form carrier symbol sequences.
- Apply differential coding to each carrier symbol sequence.
- Convert each symbol into a complex phase representation.
- Assign each carrier sequence to the appropriate IFFT bin, including the complex conjugates.

3.2.4 IFFT Operation

S.B. Weinstein [3] proposed the idea of using the FFT to separate the sub-carriers in the frequency domain. The complexity of OFDM implementation is greatly reduced with the operation and can be easily incorporated into practical systems. As the parallel sub-carriers of the signal transmitted are applied with IFFT, the spectrum is transformed to the time domain to generate one symbol period.

3.2.5 Cyclic Prefix Insertion

Wireless communications systems are susceptible to multipath channel fading. To suppress the effect of time dispersive channel, extend cyclically the duration of the OFDM symbol artificially to be longer than the impulse response of the channel. A CP is a repetition of the first section of a symbol that is appended to the end of the symbol as shown in Figure 3.3. In addition, it is important because it enables multipath representations of the original signal to fade so that they do not interfere with the subsequent symbol. As seen from the figure, the distorted part of the signal will stay within the guard interval, provided that the maximum excess delay is shorter than the length of the symbol. The guard interval thereby plays an important role in avoiding ISI.

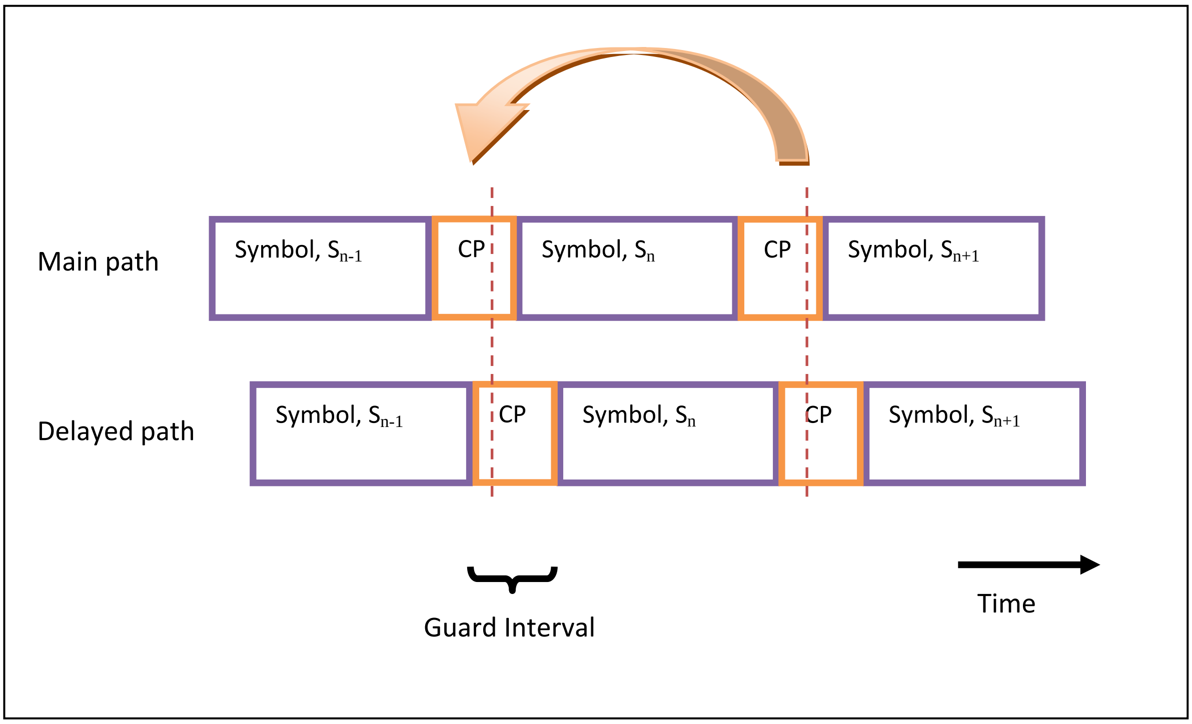

Figure 3.3 Illustration of the Cyclic Prefix insertion in an OFDM symbol

At the receiver, the samples of the cyclic extension are removed. The disadvantage of the CP is that it reduces the efficiency of the OFDM transmissions by a factor $N/(N + N_g)$, where N is the total number of sub-carriers and N_g is the length of the guard interval, or CP. This is an acceptable trade-off when considering the advantages. A guard interval length of not more than 10% of the OFDM symbol duration is employed.

3.2.6 Parallel to Serial Conversion

Once the CP has been inserted to the sub-carrier channels, they must be transmitted as one signal. Thus, the parallel to serial conversion stage is the process of summing all sub-carriers and combining them into one signal. As a result, all sub-carriers are generated perfectly simultaneously.

3.3 OFDM System Sub-Carrier Signal

Figure 3.4 shows the OFDM system sub-carrier signal, which includes a pilot-carrier (reference signal), guard sub-carrier (interference protection) and data sub-carrier (user information).

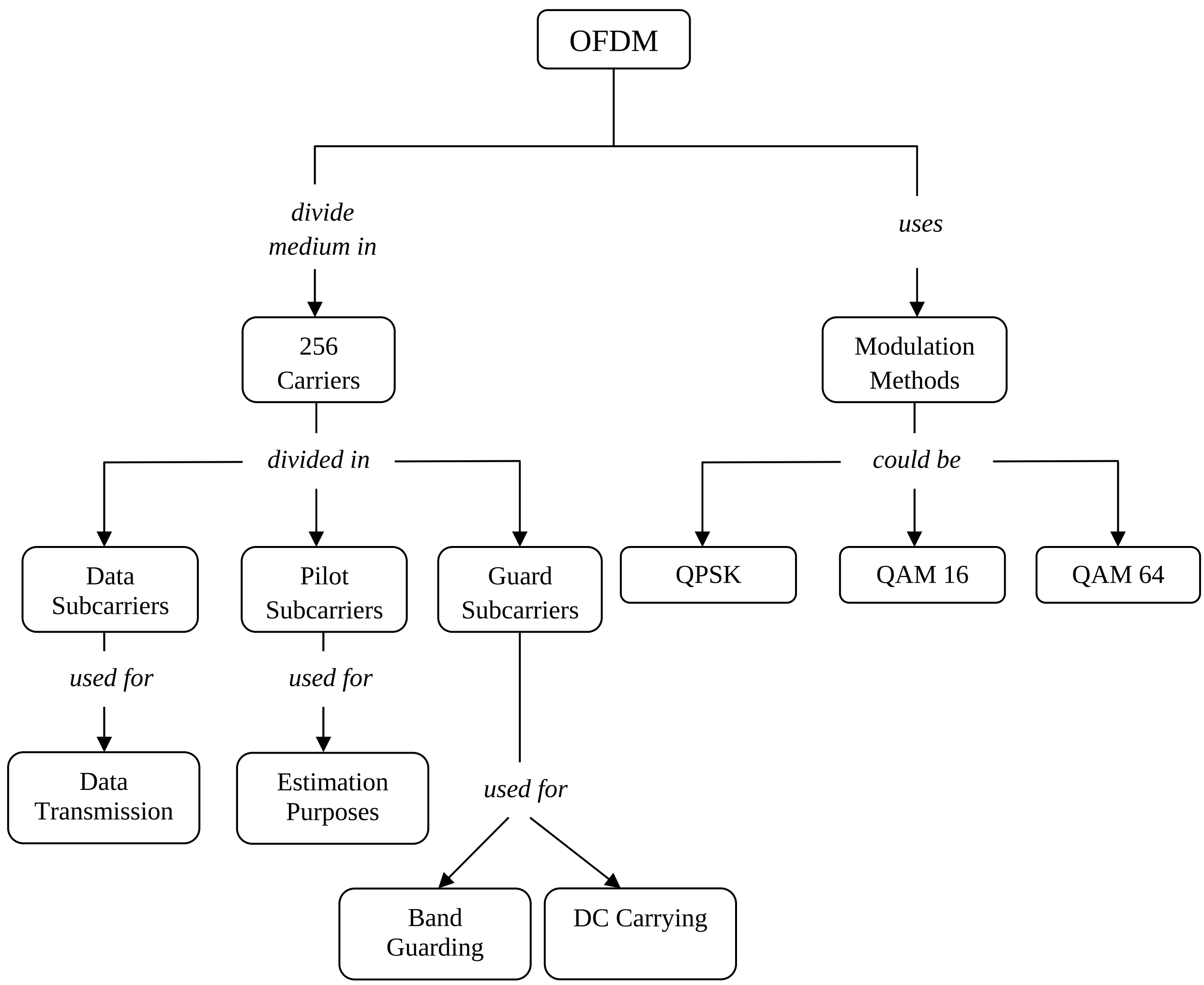

Figure 3.4 OFDM system sub-carrier signal

A sub-carrier is a modulation signal that is imposed on another carrier that can be used to independently transfer information from other sub-carriers located on the radio channel. The modulation techniques used can be QPSK, QAM 16 or QAM 64. The OFDM symbol structure is shown in Figure 3.5.

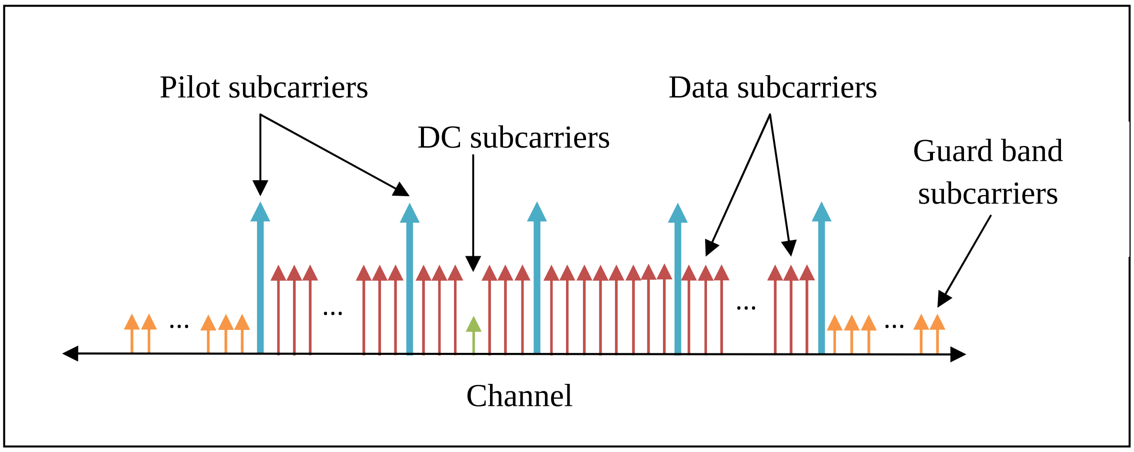

Figure 3.5 OFDM symbol structure

The symbol consists of:

(a) Pilot sub-carriers as reference signal for use in the reception of other sub-carrier signal and for various estimation purposes.

(b) Guard sub-carrier for keeping the space between OFDM signals dedicated for communication channel protection from interference.

(c) Data sub-carriers carry information of data.

(d) DC sub-carrier as the referenced from the center of the radio channel.

The maximum number of carriers used by OFDM is limited by the IFFT size. This is determined as in the following equation.

$$N_{carriers} \leq \frac{IFFTsize}{2} - 2 \qquad \text{(real-valued time signal)}$$

$$N_{carriers} \leq IFFTsize - 1 \qquad \text{(complex-valued time signal)}$$

In order to generate a real-valued time signal, OFDM (frequency) carriers are defined in complex conjugate pairs, which are symmetric about the Nyquist frequency (f_{max}). This results in the number of potential carriers to IFFT size / 2.

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